5 Polarization

5.1 Nature of polarized light

5.1.1 Linear polarization

Fig. 5.1(a) shows an electromagnetic wave with its electric field oscillating parallel to the vertical y axis. The plane containing the $\vec{E}$ vectors is called the **plane of oscillation or vibration**. We can represent the wave's **polarization** by showing the extent of the electric field oscillations in a "head-on" view of the plane of oscillation, as in Fig. 5.1 (b).

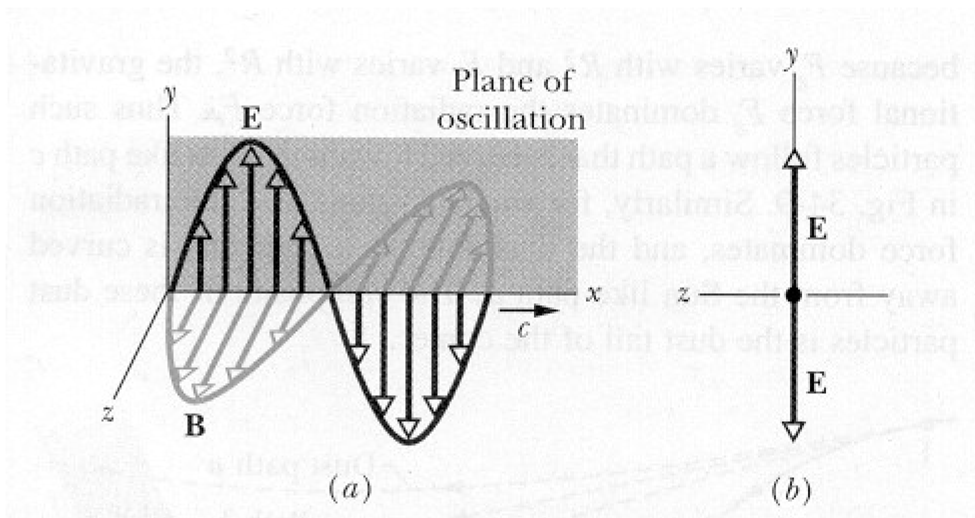


Fig. 5.1 (a) The plane of oscillation of a polarized electromagnetic wave. (b) To represent the ploarization, we view the plane of oscillation "head-on" and indicate the amplitude of the oscillating electric field.

Consider two orthogonal optical disturbances

$$\vec{E}_x(z,t) = \hat{\imath} E_{0x} \cos(kz - \omega t) \qquad (5.1)$$

and

$$\vec{E}_y(z,t) = \hat{\jmath} E_{0y} \cos(kz - \omega t + \varepsilon) \qquad (5.2)$$

where ε is the relative phase difference between the waves, both of which are traveling in the z-direction.

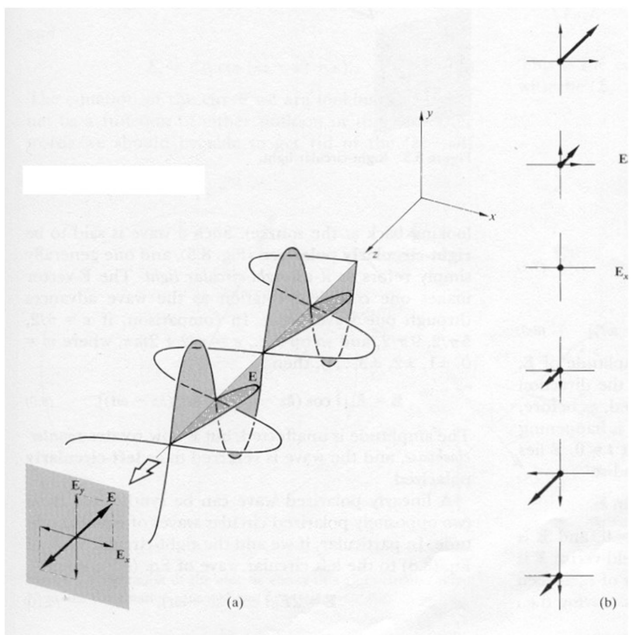


Fig. 5.2 Linear light.

The resultant optical disturbance is the vector sum of these two perpendicular waves:

$$\vec{E}(z,t) = \vec{E}_x(z,t) + \vec{E}_y(z,t). \qquad (5.3)$$

If $\varepsilon = 0$ or is an integral multiple of $\pm 2\pi$, the waves are said to be **in phase**. In that particular case Eq. 5.3 becomes

$$\vec{E}(z,t) = (\hat{i}E_{0x} + \hat{j}E_{0y})\cos(kz - \omega t). \qquad (5.4)$$

Obviously, the resultant wave is also linearly polarized, as shown in Fig. 5.2. A single resultant electric-field oscillates, along a tilted line, consinusoidally in time [Fig. 5.2 (b)].

This process of addition can be carried out equally well in reverse; that is, we resolve any plane-polarized wave into two orthogonal components.

5.1.2 Circular polarization

Now we consider another particular case. That is, $E_{0x} = E_{0y} = E_0$, in addition, $\varepsilon = -\pi/2 + 2m\pi$, where $m = 0, \pm 1, \pm 2, \cdots$. Accordingly,

$$\vec{E}_x(z,t) = \hat{i}E_0\cos(kz - \omega t) \qquad (5.5)$$

and

$$\vec{E}_y(z,t) = \hat{j}E_0 \sin(kz - \omega t). \qquad (5.6)$$

The resultant wave is given by

$$\vec{E} = E_0[\hat{i}\cos(kz - \omega t) + \hat{j}\sin(kz - \omega t)], \qquad (5.7)$$

as shown in Fig. 5.3.

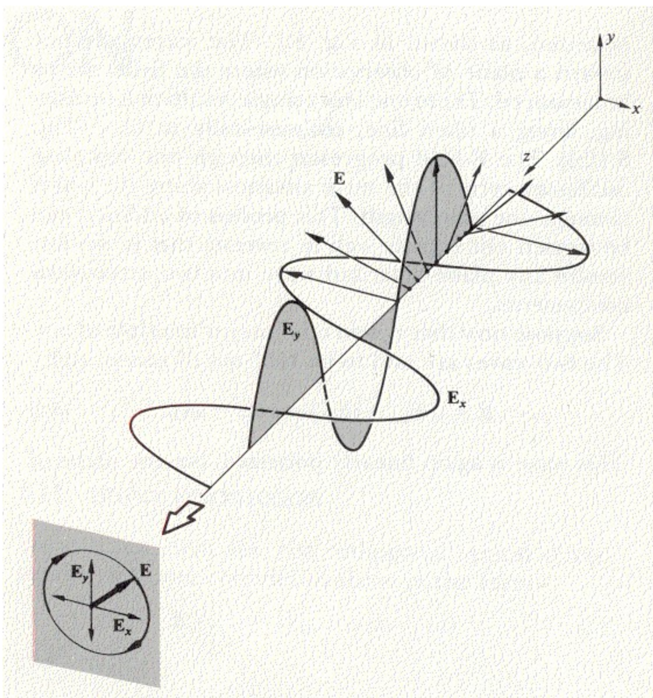


Fig 5.3 Right-circular light.

The resultant electric field vector $\vec{E}$ is rotating clockwise at an angular frequency of ω. Such a wave is said to be **right-circularly polarized**. In comparison, if $\varepsilon = \pi/2 + 2m\pi$, where $m = 0, \ \pm 1, \ \pm 2, \cdots,$ then

$$\vec{E} = E_0[\hat{i}\cos(kz - \omega t) - \hat{j}\sin(kz - \omega t)]. \qquad (5.8)$$

The amplitude is unaffected, but $\vec{E}$ now rotates counter-clockwise, and the wave is referred to as **left-circularly polarized**.

A linearly polarized wave can be synthesized from two oppositely polarized circular waves of equal amplitude. In particular, if we add Eq. 5.7 to Eq. 5.8, we get a linearly polarized wave,

$$\vec{E} = 2E_0\hat{i}\cos(kz - \omega t). \qquad (5.9)$$

5.1.3 Elliptical polarization

Now we consider a general case. Recall that

$$E_x = E_{0x}\cos(kz - \omega t) \qquad (5.10)$$

and

$$E_y = E_{0y}\cos(kz - \omega t + \varepsilon). \qquad (5.11)$$

Expand the expression for E_y into

$$E_y / E_{0y} = \cos(kz - \omega t)\cos\varepsilon - \sin(kz - \omega t)\sin\varepsilon$$

and combine it with $E_x / E_{0x} = \cos(kz - \omega t)$ to yield

$$\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}}\cos\varepsilon = -\sin(kz - \omega t)\sin\varepsilon. \qquad (5.12)$$

It follows from Eq. 5.10 that

$$\sin(kz - \omega t) = [1 - (E_x / E_{0x})^2]^{1/2},$$

So Eq. 5.12 leads to

$$\left(\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}}\cos\varepsilon\right)^2 = \left[1 - \left(\frac{E_x}{E_{0x}}\right)^2\right]\sin^2\varepsilon.$$

Finally, on rearranging terms, we have

$$\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 - 2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\varepsilon = \sin^2\varepsilon.. \qquad (5.13)$$

This is the equation of an ellipse making an angle α with the (E_x, E_y)-coordinate system (Fig. 5.4) such that

$$\tan 2\alpha = \frac{2E_{0x}E_{0y}\cos\varepsilon}{E_{0x}^2 - E_{0y}^2}. \qquad (5.14)$$

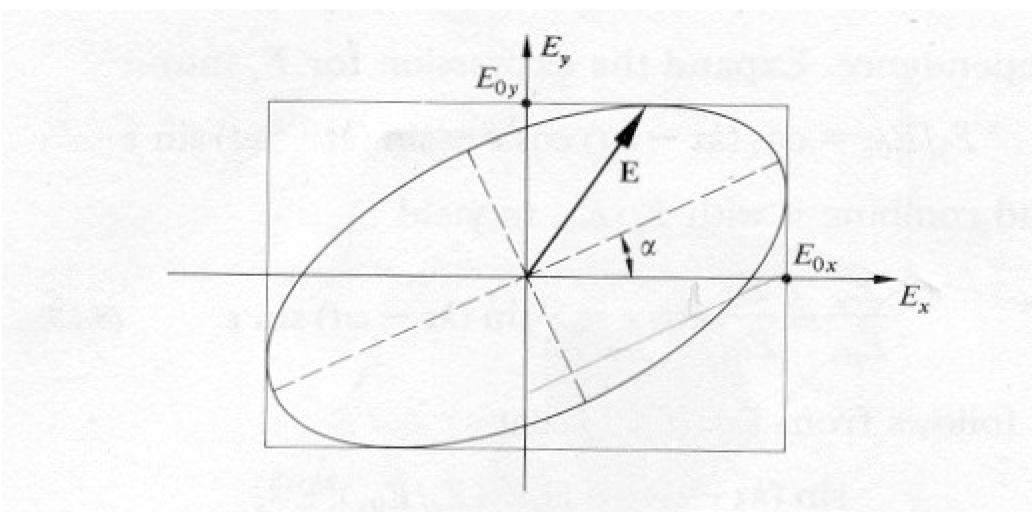


Fig. 5.4 Elliptical light.

If $\alpha = 0$ or equivalently $\varepsilon = \pm\pi/2,\ \pm 3\pi/2,\ \pm 5\pi/2, \cdots,$ we have familiar form

$$\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 = 1. \qquad (5.15)$$

Furthermore, if $E_{0y} = E_{0x} = E_0$, this can be reduced to

$$E_y^2 + E_x^2 = E_0^2. \qquad (5.16)$$

Clearly, it is a circle.

If ε is an even multiple of π, Eq. 5.13 results in

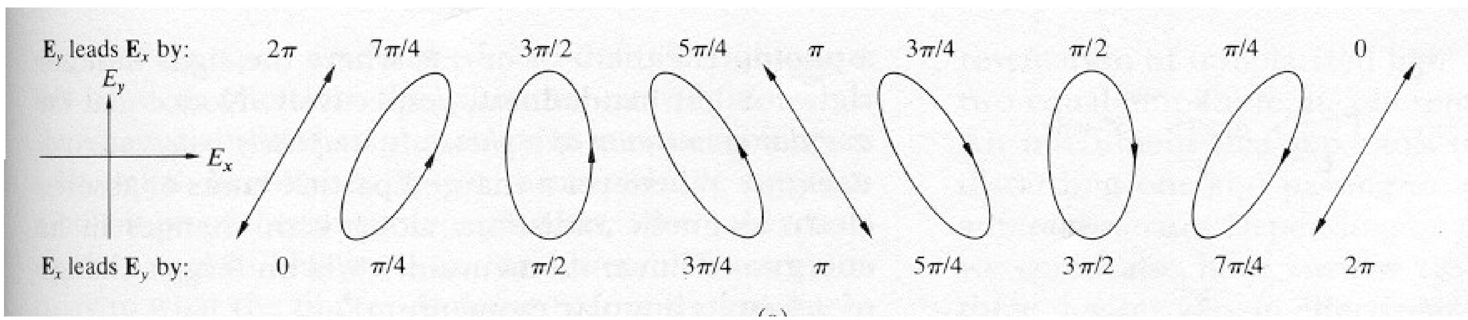
$$E_y = \frac{E_{0y}}{E_{0x}} E_x \qquad (5.17)$$

And similarly for odd multiples of π,

$$E_y = -\frac{E_{0y}}{E_{0x}} E_x. \qquad (5.18)$$

These are both straight lines having slopes of $\pm E_{0y}/E_{0x}$; in other words, we have linear light. So, both linear and circular light may be considered to be special cases of **elliptically polarized** light.

Fig. 5.5 gives various polarization configurations. This very important diagram is labeled across the bottom "E_x leads by E_y :0, $\pi/4$, $\pi/2$, $3\pi/4, \cdots,$" where these are the positive values of ε to be used in Eq. 5.2.



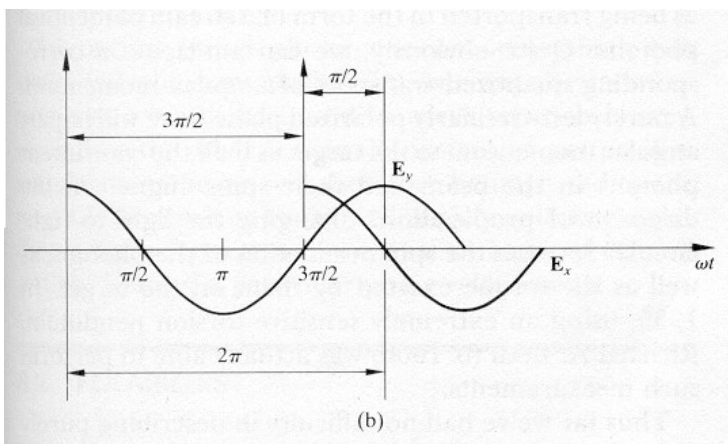


Fig. 5.5 (a) Various polarization configurations. (b) E_x leads E_y by $\pi/2$, or alternatively, E_y leads E_x by $3\pi/2$.